the divergence is not high, comprising  $4.5 \cdot 10^{-6}$  deg K/bar at the lowest temperature presented in [2], 2.74°C. All the divergences are significantly less than those noted in [1] for values calculated with equations of state, which again illustrates the fact that in the liquid region equations of state based on P, v, T data which describe density well can lead to significantly diverging values of derivative properties.

We note that in our calculation experimental data on density and isobaric heat capacity are necessary only at atmospheric pressure. For the high-pressure region only data on the speed of sound are required [4]. The results obtained indicate the possibility of accurate calculation of  $\beta_{\rm S}$  from acoustical studies without performing complex experimental measurements of this quantity.

## NOTATION

 $\beta_s$ , temperature coefficient of adiabatic compression; T, absolute temperature;  $C_p$ , isobaric heat capacity; v, specific volume; s, entropy; p, pressure.

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VOLT-AMPERE CHARACTERISTICS OF AN AC ARC IN A TRANSVERSE GAS FLOW

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UDC 537.523.5

An analytical dependence of the volt-ampere characteristic of an arc on the velocity of the incoming transverse gas flow is obtained. The stability threshold of arc combustion is determined.

The ac electric arc is widely used as an intense heat source in various devices: steelsmelting furnaces, welding instruments, plasmotrons, etc. The existing theoretical investigations in the literature are basically calculations of an arc without a gas flow [1] and with a cotraveling flow [2]. There are practically no data on an arc in a transverse flow. Only in [3] was the experimental volt-ampere characteristic (VAC) of an arc moving in air under the action of a rotating magnetic field investigated.

In deriving the theoretical equations for an arc in a transverse gas flow, the following assumptions are made: the form of the arc is cylindrical; the axis 0x is parallel to the direction of the incoming flow; the axis 0z coincides with the arc axis; the influence of the intrinsic magnetic field of the arc and the pressure gradients in the gas flow is neglected; processes occurring close to the electrodes are neglected; a constant mass velocity  $\vec{u} = \rho \vec{v}$  is assumed inside the arc; and the pulsations of enthalpy and gas velocity following the arc are not taken into account.

First consider the enthalpy distribution in the arc. The energy equation for the given arc model is written in the form

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$$\rho \frac{\partial h}{\partial t} + (\vec{u}, \nabla h) = \nabla (\kappa \nabla \Theta) + \sigma E^2 - Q$$
<sup>(1)</sup>

with the conditions  $\sigma = 0$  and  $h = h_1$  at the arc boundary  $(r = r_1)$ .

With linear approximation of the dependences of the gas properties

$$M = \overline{Mh}; \quad N = \overline{Nh}; \quad \sigma = \overline{\sigmah}; \quad Q = \overline{Qh},$$

where

$$M = \int_{h_1}^{h} \rho dh; \ N = \int_{\Theta_1}^{\Theta} x d\Theta; \ \bar{h} = \frac{h - h_1}{h_1}$$

the energy Eq. (1) takes the form

$$\frac{\partial \bar{h}}{\partial t} + (\bar{w}, \ \nabla \bar{h}) = a^2 \nabla^2 \bar{h} + (\varepsilon^2 - \bar{\varepsilon}^2) \bar{h}, \tag{2}$$

where  $\vec{w} = \vec{h_1 u} / \vec{M}$ ;  $a^2 = \vec{N} / \vec{M}$ ;  $\varepsilon^2(t) = \vec{\sigma} E^2(t) / \vec{M}$ ;  $\vec{\varepsilon}^2 = \vec{Q} / \vec{M}$ .

The result of solving Eq. (2) with the boundary condition  $\bar{h}(r_1, t) = 0$  is

$$\bar{h}(r, t) = AJ_0\left(\mu_1 \frac{r}{r_1}\right) \exp\left(\frac{(\vec{w}, \vec{r})}{2a^2} + \int_0^t \left(\varepsilon^2 - \overline{\varepsilon^2} - \frac{w^2}{4a^2} - \frac{\mu_1^2 a^2}{r_1^2}\right) dt\right),$$
(3)

where A = const;  $\mu_1$  is the first root of the equation  $J_0(x) = 0$ .

In order to satisfy the periodicity condition  $\overline{h}(\mathbf{r}, \mathbf{t}) = \overline{h}(\mathbf{r}, \mathbf{t} + \mathbf{T})$ , it is necessary to set  $\int_{1}^{T} \left( \varepsilon^{2} - \overline{\varepsilon^{2}} - \frac{w^{2}}{4a^{2}} - \frac{\mu_{1}^{2}a^{2}}{r_{1}^{2}} \right) dt = 0.$ 

Introducing the notation

$$\frac{1}{T}\int_{0}^{T}\varepsilon^{2}dt=\varepsilon_{\mathrm{E}}^{2}$$

the periodicity condition is written in the form

$$\varepsilon_{\rm E}^2 = \overline{\varepsilon}^2 + \frac{\omega^2}{4a^2} + \frac{\mu_1^2 a^2}{r_1^2} \,. \tag{4}$$

The current strength  $i = 2\pi \int_{0}^{r_{i}} \sigma Erdr$  in the arc is determined using Eqs. (3) and (4)

$$i(t) = \frac{2\pi A r_1^2 \mu_1 (\overline{M}\overline{\sigma})^{1/2}}{\mu_1^2 + \frac{w^2 r_1^2}{4a^4}} I_0 \left(\frac{wr_1}{2a^2}\right) J_1(\mu_1) \quad \varepsilon(t) \exp\left(\int_0^t (\varepsilon^2 - \varepsilon_{\rm E}^2) dt\right), \tag{5}$$

where  $I_0$  is the modified Bessel function and the constant A is expressed in terms of the effective value of the current strength

$$\mathbf{I}_{\mathbf{E}} = \left(\frac{1}{T} \int_{0}^{T} i^{2}(t) dt\right)^{1/2}$$

The only difference between Eq. (5) and the analogous relation in [1] is the form of the constant factor preceding the combination

$$\varepsilon(t) \exp\left(\int_{0}^{t} (\varepsilon^{2} - \varepsilon_{\mathbf{E}}^{2}) dt\right).$$

In contrast to [1], the investigation is not of the time dependences E(t) and i(t) in the arc included in a particular circuit, but rather of the influence of the gas-flow velocity on the characteristic  $E_E(I_E)$ .

Since Eq. (5) includes the undetermined radius  $r_1$ , it is not sufficient for finding this dependence. To eliminate  $r_1$ , the heat extraction from the arc by the incoming gas flow is considered. The gas flow beyond the limits of the arc  $(x > r_1)$  is described by a system of boundary-layer equations

$$\rho\left(v_x \frac{\partial h}{\partial x} + v_y \frac{\partial h}{\partial y}\right) = \frac{\partial}{\partial y} \left(v \rho \frac{\partial h}{\partial y}\right), \qquad (6)$$

$$\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = \frac{\partial}{\partial y} \left(v \rho \frac{\partial v_x}{\partial y}\right), \tag{7}$$

$$\operatorname{liv}\rho v = 0, \tag{8}$$

$$\rho h \sim \text{const},$$
 (9)

setting  $\sigma = 0$ , Q = 0,  $v\rho/h = const$ , Pr = 1. The following boundary conditions are added:  $v_{x}(\infty) = v_{0}, v_{y}(\infty) = 0, h(\infty) = h_{0}, \rho(\infty) = \rho_{0}, h(r_{1}, 0) = h_{1}, (\partial v_{x}/\partial y)|_{y=0} = 0, (\partial h/\partial y)|_{y=0} = 0.$ 

The solution of Eqs. (6)-(9) for the gas flow is first found far from the arc, where  $v_x - v_0 \ll v_0$ . The next step is to move from the variables x and y to the Dorodnitsyn variables  $\xi$ ,  $\eta$  and then to the Mises variables  $\xi$  and  $\varphi$  [4].

Integration of Eqs. (6) and (7) gives

$$h = h_0 + \frac{B}{\xi^{1/2}} \exp\left(-\frac{\varphi^2}{4v_0 v_0 \xi}\right),$$
 (10)

$$v_{x} = v_{0} + \frac{C}{\xi^{1/2}} \exp\left(-\frac{\varphi^{2}}{4v_{0}v_{0}\xi}\right).$$
(11)

The constant C is expressed in terms of the heat flux from unit length of the arc discharge  $q = \oint \rho h v_n ds$ , where the integral  $\oint$  is taken over a closed contour covering the arc cross section; vn is the gas-velocity component normal to the contour; s is the contour length. Choosing the integration contour in the form of two infinite straight lines  $\xi =$ const, one of which is taken far in front of, and the other far behind, the arc, and taking into account that  $\varphi = v_0 y$  far from the arc, it is found that

$$q = \rho_0 h_0 \int_{-\infty}^{\infty} (v_x - v_0) \, dy$$

Hence, using Eq. (11), it is found that

$$C = \frac{q}{2\rho_0 h_0} \sqrt{\frac{v_0}{\pi v_0}}$$
 (12)

The quantity q is found using Eq. (3), averaged over the period

$$q = \frac{I_{\rm E} \left(\epsilon_{\rm E}^2 - \bar{\epsilon}^2\right) \overline{N} \alpha}{a^2 \epsilon_{\rm F} \left(\overline{M \, \sigma}\right)^{1/2}},$$
(13)

where

$$\alpha = \frac{\left\langle \exp\left(\int_{0}^{t} (\varepsilon^{2} - \varepsilon_{E}^{2}) dt\right) \right\rangle}{\left\langle \exp\left(2\int_{0}^{t} (\varepsilon^{2} - \varepsilon_{E}^{2}) dt\right) \right\rangle^{1/2}}; \quad \langle f(t) \rangle = \frac{1}{T} \int_{0}^{T} f(t) dt.$$

The constant B in Eq. (10) is found from Eqs. (8), (9), and (11) and the identity

$$\operatorname{div} \vec{pv} = \rho_0 h_0 \left( \frac{1}{h} \operatorname{div} \vec{v} - \frac{(\vec{v}, \nabla h)}{h^2} \right),$$
$$B = \frac{h_0}{v_0} C.$$

(14)

and hence

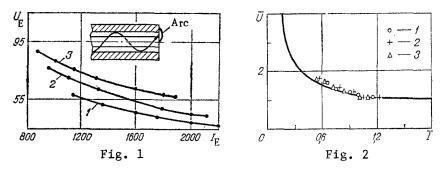


Fig. 1. Dependence of the potential at the arc  $U_E$  (V) on the current strength  $I_E$  (A) in a transverse air flow: 1) G = 1.7  $10^{-3}$ ; 2) 2.5  $10^{-3}$ ; 3) 2.9  $10^{-3}$  kg/sec; d = 0.065 m; L = 0.15 m.

Fig. 2. Comparison of the theoretical and experimental VAC: 1-3) as in Fig. 1; the curve corresponds to calculation from Eq. (17).

Assuming that the velocity  $v_x$  changes slowly even close to the arc, Eq. (10) is used to determine the arc radius  $r_1$ . When  $\phi = 0$  and  $\xi = r_1$ , it is found that

$$r_1 = \frac{B^2}{(h_1 - h_0)^2}.$$
(15)

Using Eqs. (4) and (12)-(15), it is possible to find the relation between  $E_E$  and  $I_E$ , i.e., the VAC equation

$$\frac{I_{\rm E}(E_{\rm E}^2-b^2)}{E_{\rm F}} = \frac{(Dv_0)^{1/2}}{(E_{\rm F}^2-b^2-Fv_0^2)^{1/4}},$$
(16)

where  $D = 4\pi\rho_0^2 v_0 \mu_1 \overline{N}^{1/2} (h_1 - h_0)^2 / \alpha^2 \overline{\sigma}^{1/2}$ ;  $b^2 = \overline{Q}/\overline{\sigma}$ ;  $F = (\delta\rho_0 h_1)^2 / 4\overline{N\sigma}$ ;  $\delta = u/\rho_0 v_0$ . The coefficient  $\delta$  is found in the limits  $0 < \delta < 1$ , and takes account of the gas expansion before entering the arc. It follows from Eq. (16) that the VAC of an ac arc in a transverse gas flow is descending; with increase in I<sub>E</sub>, the field strength in the arc tends to a limiting value  $E_{\min} = (b^2 + Fv_0^2)^{1/2}$ .

With increase in the velocity  $v_0$ , the field strength in the arc increases. This means that a sufficiently strong gas flow may "switch off" the arc. In fact, if the field strength in the arc is bounded by the value  $E_s$  corresponding to the current source, increase in the gas value above the value  $[(E_s^2 - b^2)/F]^{1/2}$  leads to loss of the solution of Eq. (16). In practice, this case corresponds to quenching of the arc with excessive increase in gas flow rate.

It is interesting to compare the theoretical VAC with the experimental curve obtained for an ac plasmotron with graphite electrons [5]. In this plasmotron, an arc of selfstabilizing length burns at the ends of rod and ring electrons and is in a transverse swirling air flow, as shown in Fig. 1. Neglecting the arc radiation (b = 0) and passing from the field strength  $E_E$  to the potential at the arc  $U_E$ , Eq. (16) is written in dimensionless form

$$\overline{U}\sqrt{\overline{U^2}-1} = 1/\overline{I},\tag{17}$$

where  $\overline{U} = U_E/U_{min}$ ;  $\overline{I} = I_E/I_e$ ;  $I_e = l(lDv_o)^{1/2}/U_{min}^{3/2}$ ;  $U_{min} = lF^{1/2}v_o$ . It is readily evident that, when  $I_E = I_e$ , the dimensionless characteristic potential  $\overline{U}_e = 1.21$ .

In analyzing the experimental data on the VAC for the given plasmotron, the sum of the near-electrode potential drops — equal to  $\sim$ 15 V according to the estimates of [6] — is sub-tracted from the measured potential at the electrodes. Then the characteristic current strength I<sub>e</sub> and U<sub>e</sub> = 1.21U<sub>min</sub> is found, where U<sub>min</sub> is the minimum potential at the arc, determined by extrapolating the experimental curves of U<sub>E</sub> = f(I<sub>E</sub>) to the large-current region.

It is evident from Fig. 2 that theory and experiment are in fair agreement for currents  $\overline{I} = 0.5-1.3$  (I<sub>E</sub> = 800-2200 A). Unfortunately, unstable conditions of plasmotron operation at small currents (I<sub>E</sub> < 800 A) and the limits on the power-source capacity mean that experimental testing cannot be performed for other sections of the theoretical VAC.

All the basic conclusions obtained for an ac arc in a transverse gas flow are also valid for a dc arc; all that is required is to set  $\partial h/\partial t = 0$  in Eq. (1).

## NOTATION

h, enthalpy;  $\rho$ , density; t, time; v, velocity; x,  $\sigma$ , thermal conductivity and electrical conductivity; E, electric field strength; U, potential at the arc; Q, radiant flux; T, period of oscillation of the current; r, radius; l, length of electric arc; d, L, diameter and length of electrode;  $\Theta$ , temperature; v, kinematic viscosity; Pr, Prandtl number;  $\varphi$ , current function. Indices: 0, gas parameters far from arc; 1, parameters at arc boundary; x, projection on the axis 0x; y, projection on the axis 0y; E, effective value; e, characteristic value.

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OPTIMIZATION OF METAL HEATING IN PLASMA-MECHANICAL TREATMENT

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Results are presented of an experimental investigation of the process of energy transfer to the anode of an arc in application to plasma-mechanical treatment of components.

The process of cutting ultrastrong brands of steel is characterized by large thermal and force loads on the cutting instrument. Consequently, its sturdiness is reduced, the time occupied in treating one component is increased, and the power, size, and weight of the tools grow.

The recently proposed process of plasma-mechanical treatment (PMT) permits raising the weight productivity of the cutting process, i.e., the mass of the chips removed from the workpiece in unit time. PMT is a combination method of molding the component that includes heating the layer to be cut by a high-current stabilized arc (HSA) and its subsequent removal by the cutting instrument [1]. As a rule the component to be treated is the anode. In certain cases it is expedient to submelt part of the metal and blow it off by plasma jets.

Utilization of the HSA has the following advantages over other kinds of heating: the high space-time stability of the arc column and the near-anode domain, the high energy flux concentration on the surface of the material being treated, the convenience of regulating the power of the plasma flux.

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